ANSWER SHEET

1	(B)	13	(1569 to 1571)	25	(250)	37	(*)	49	(1.67 to 1.69)	61	(D)
2	(795 to 805)	14	(B)	26	(92 to 95)	38	(6.60 to 6.65)	50	(C)	62	(D)
3	(D)	15	(D)	27	(D)	39	(A)	51	(B)	63	(C)
4	(B)	16	(C)	28	(B)	40	(a^4/5)	52	(3)	64	(D)
5	(B)	17	(C)	29	(D)	41	(4400 to 4600)	53	(6)	65	(D)
6	(B)	18	(D)	30	(D)	42	(A)	54	(0.10 to 0.11)		
7	(B)	19	(C)	31	(D)	43	(490000 to 510000)	55	(4)		
8	(13.5 to 14.5)	20	(29 to 31)	32	(B)	44	(0.30 to 0.35)	56	(A)		
9	(B)	21	(159 to 161)	33	(A)	45	(B)	57	(B)		
10	(D)	22	(A)	34	(B)	46	(1079 to 1081)	58	(B)		
11	(C)	23	(0.34 to 0.36)	35	(15 to 18)	47	(1.75 to 1.85)	59	(B)		
12	(C)	24	(D)	36	(0.095 to 0.115)	48	(A)	60	(D)		

SOLUTION

1. (B)

The elongation of bar due to its own weight (W) is

$$\Delta = \frac{WL}{2AE}$$

$$\therefore \quad \Delta = \frac{\gamma L^2}{2E}$$

Where, $\gamma = \frac{\text{Weight}}{\text{Volume}}$

2. (795 to 805)

$$\text{Strain Tensor} = \begin{bmatrix} \epsilon_{xx} & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \epsilon_{yy} & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \epsilon_{zz} \end{bmatrix}$$

On comparison, we get

$$\frac{\gamma_{xy}}{2} = 0.004$$

$$\Rightarrow \quad \gamma_{xy} = 0.008$$

$$G = \frac{\tau_{xy}}{\gamma_{xy}}$$

$$\Rightarrow \quad 100 \times 10^3 = \frac{\tau_{xy}}{0.008}$$

$$\tau_{xy} = 800\, MPa$$

3. (D)

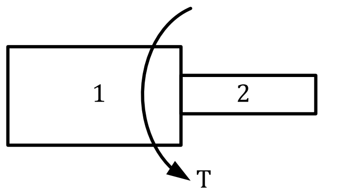
Torque on part BC = T

Torque on part AC = 2T

$$U = \frac{T^2 (L/2)}{2GI_P} + \frac{(2T)^2 (L/2)}{2GI_P}$$

$$= \frac{T^2 L}{4GI_P} + \frac{4T^2 L}{4GI_P} = \frac{5T^2 L}{4GI_P}$$

4. (B)



$\theta_1 = \theta_2$ and T gets apportioned as T_1 and T_2

5. (B)

6. (B)

Vertical reaction at fixed end

$$F = \frac{1}{2} \times 100 \times 1 = 50\, N$$

Bending moment at fixed end.

$$M = \frac{100 \times 1}{2} \times \left(1 + \frac{2}{3} \times 1\right) = \frac{250}{3} N - m$$

7. (B)

$$\begin{cases} \epsilon_1 = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} \\ \epsilon_2 = \frac{\sigma_2}{E} - \frac{\mu \sigma_1}{E} \end{cases}$$

$$E \times 800 \times 10^{-6} = \sigma_1 - \frac{\sigma_2}{4} \qquad \text{...(i)}$$

$$E\times\left(-600\times10^{-6}\right)=\pm_2-\frac{\pm_1}{4} \quad \text{...(ii)}$$

Solving equations (i) and (ii), we get

$\pm_1 = 138.67\,MPa$

And $\pm_2 = -85.33\,MPa$

8. (13.5 to 14.5)

$\pm_x = 30\,MPa$

$\pm_y = 18\,MPa$

$²_{xy} = 8\,MPa$

$$\pm_2=\left(\frac{\pm_x+\pm_y}{2}\right)-\sqrt{\left(\frac{\pm_x-\pm_y}{2}\right)^2+²^2_{xy}}$$

$$\pm_2=\left(\frac{30+18}{2}\right)-\sqrt{\left(\frac{30-18}{2}\right)^2+8^2}$$

$$\pm_2=24-\sqrt{36+64}$$

$$\pm_2=24-10=14\,MPa$$

Alternatively,

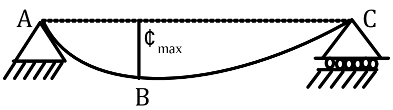
Summation of normal stresses on two mutually perpendicular planes is always constant i.e.

$\pm_x+\pm_y=\pm_1+\pm_2$

$\Rightarrow \quad 30+18=34+\pm_2$

$\Rightarrow \quad \pm_2=14\,MPa$

9. (B)



10. (D)

Both the ends will behave as fixed supports in this case because K_T approaches infinity.

$$\therefore \quad P_{cr}=\frac{4®^2EI}{L^2}=\frac{\sim®^2EI}{L^2}$$

$\Rightarrow \quad \sim = 4.0$

11. (C)

Slenderness ratio, $ª=\frac{L_{eff}}{r}$

Where r is the least radius of gyration and L_{eff} is effective length

$$r=\sqrt{\frac{I}{A}}$$

$$r=\frac{d}{4}$$

$$\therefore \quad \frac{L_{eff}}{d}=\frac{160}{4}=40$$

12. (C)

Angular twist in spring is

$$– = \frac{32WRL}{®d^4G}$$

Where,

d is diameter of wire of spring

G is modulus of rigidity

W is axial pull on spring

R is mean radius of coil

L is length of spring

$$\therefore \quad \frac{–_2}{–_1}=\frac{W_2}{W_1}\times\frac{L_2}{L_1}$$

$$=\frac{W/2}{W}\times\frac{L/2}{L}=\frac{1}{4}$$

13. (1569 to 1571)

Wire diameter, $d=10\,mm$

Spring index $=5=\frac{D}{d}$

$D=5d$

$D=50\,mm$

Number of turns $n=10$

Length of wire $=®D\times n$

$=®\times50\times10$

$=3.14\times500$

$=1570\,mm$

14. (B)

15. (D)

16. (C)

For one end fixed other free

$(P_{cr})_1 \propto \frac{1}{(2l)^2} \qquad (P_{cr})_1=\frac{K}{4l^2}$

$$(P_{cr})_2=\frac{K}{\left(\frac{l}{\sqrt{2}}\right)^2}=\frac{2k}{l^2}$$

$$\frac{(P_{cr})_2}{(P_{cr})_1}=\frac{2\frac{K}{l^2}}{\frac{K}{4l^2}}$$

$$(P_{cr})_2=8(P_{cr})_1$$

17. (C)

18. (D)

If T be tension in string BC and since it passes over smooth pulley C,

$T = W_1$.

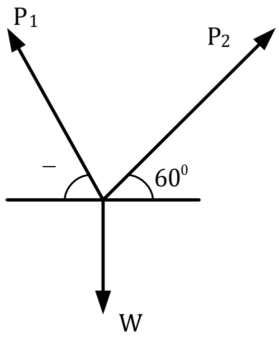
Reaction at B is $\frac{W}{2}$,

$\therefore\ \frac{W}{2} = T\cos 45^0 = W_1 \times \frac{1}{\sqrt{2}}$

Or $W_1 = \frac{W\sqrt{2}}{2} = \frac{W}{\sqrt{2}}$

19. (C)

20. (29 to 31)



$\Sigma F_x = 0$

$-P_1\cos\theta + P_2\cos 60 = 0$ ...(i)

$\Sigma F_y = 0$

$P_1\sin\theta + P_2\sin 60 = W$ ...(ii)

$\Rightarrow\ P_2 = \frac{1}{\sin 60}\left[W - P_1\sin\theta\right]$ ...(iii)

$\Rightarrow\ P_2 = \frac{2}{\sqrt{3}}\left[W - P_1\sin\theta\right]$ ...(iv)

$\Rightarrow\ -P_1\cos\theta + \frac{2}{\sqrt{3}}\left[W - P_1\sin\theta\right]\cos 60^0 = 0$ (Putting the value of P_2 from equation (iv) in equation (i))

$\Rightarrow\ -P_1\cos\theta + \frac{2}{\sqrt{3}} \times \frac{1}{2}\left[W - P_1\sin\theta\right] = 0$

$\Rightarrow\ -P_1\cos\theta - \frac{P_1}{\sqrt{3}}\sin\theta = -\frac{W}{\sqrt{3}}$

$\Rightarrow\ P_1\cos\theta + \frac{P_1\sin\theta}{\sqrt{3}} = +\frac{W}{\sqrt{3}}$

$\Rightarrow\ \left[\sqrt{3}\cos\theta + \sin\theta\right]P_1 = W$

$P_1 = \frac{W}{\sqrt{3}\cos\theta + \sin\theta}$

For P_1 to be minimum denominator should be maximum

$\frac{d}{d\theta}\left[\sqrt{3}\cos\theta + \sin\theta\right] = 0$

$\Rightarrow\ \sqrt{3}\sin\theta + \cos\theta = 0$

$\Rightarrow\ \tan\theta = \frac{1}{\sqrt{3}}$

$\boxed{\theta = 30^0}$

21. (159 to 161)

Range = $\frac{2u^2\sin 2\theta}{g}$

Given, Maximum height $= 40m = \frac{u^2}{2g}$

$\Rightarrow\ u^2 = 2 \times 9.81 \times 40$

$\Rightarrow\ u = 28\,m/\text{sec}$

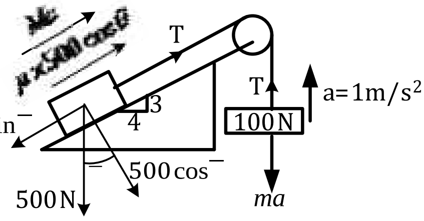
$\therefore$ Range $= \frac{2 \times 28^2 \times \sin(2 \times 45)}{9.81}$

$= 160\,m$

22. (A)

Disc is thin hence point contact

23. (0.34 to 0.36)



$T - mg = ma$

$T - 100 = \left(\frac{100}{g}\right)a$

$T = 100 + (10 \times 1) = 110\,N$ ...(i)

Now, $Mg\sin\theta - \mu_k Mg\cos\theta - T = Ma$

$\frac{500}{g} \times g \times \frac{3}{5} - \mu_k \frac{500}{g} \times g \times \frac{4}{5} - 110 = \left(\frac{500}{g}\right) \times 1$

$300 - \mu_k \times 400 - 110 = 50$

$\boxed{\mu_k = 0.35}$

24. (D)

$\frac{m_1}{m_2} = \frac{1}{4}$

$KE_1 = KE_2$

$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2$

$\Rightarrow\ \frac{m_1}{m_2} = \frac{v_2^2}{v_1^2}$

$\Rightarrow\ \frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

Now $\frac{m_1v_1}{m_2v_2} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$

25. (250)

$\mu_2 = \mu_1 + \sim t$

$\Rightarrow \quad 0 = \frac{3000}{60} + \sim \times 10$

$\boxed{\sim\ = -5\,rad/\sec^2}$

$- = \mu_1 t + \frac{1}{2}\sim t^2$

$\Rightarrow \quad - = 500 + \frac{1}{2}(-5)\times 100$

250 revolutions

26. (92 to 95)

$¢ = \frac{WL^3}{3EI}$

$- = \frac{WL^2}{2EI}$

$¢ = \left(\frac{WL^2}{2EI}\right)\left(\frac{2L}{3}\right)$

$¢ = - \times \frac{2L}{3} = \frac{0.035\times 2\times 4}{3}$

$= 0.0933\,m\,or\,93.3\,mm$

27. (D)

BMD will be of 2nd degree, so the correct answer is (D)

28. (B)

Vertical reaction at fixed end

$F = \frac{1}{2}\times 100\times 1 = 50\,N$

Bending moment at fixed end.

$M = \frac{100\times 1}{2}\times\left(1+\frac{2}{3}\times 1\right) = \frac{250}{3}N-m$

29. (D)

$^2 \propto r$

30. (D)

After equating central deflection for both the cases, we have

$\frac{5}{384}\frac{WL^3}{EI} = \frac{RL^3}{48EI}$

$\Rightarrow \quad R = \frac{5W}{8}$

31. (D)

$\frac{\pm}{y} = \frac{E}{R} = \frac{M}{I}$ (Bending formula)

$y = \frac{0.2}{2}mm = 0.1\,mm$

$R = \frac{25}{2} = 12.5\,mm$

$\pm_{max} = \frac{100\times 0.1}{12.5} = 0.8\,GPa$

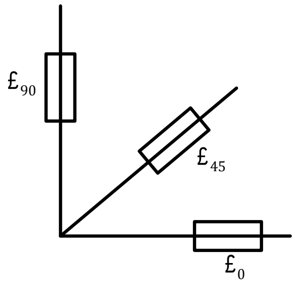
$= 0.8\times 10^3\,MPa = 800\,MPa$

32. (B)

The symmetry of the shaft shows that there is no torsion on section AB.

$\therefore$ Rotation, $-_1 = \frac{TL}{GJ_1}$

33. (A)



Rectangular strain gauge rosette

$£_{0^0} = -220\times 10^{-6}$

$£_{45^0} = 120\times 10^{-6}$

$£_{90^0} = 220\times 10^{-6}$

$E = 2\times 10^5\,N/mm^2$

$« = 0.3$

Now, principal strains are given by

$£_{1,2} = \frac{£_0 + £_{45}}{2} \pm \frac{1}{\sqrt{2}}\sqrt{\left(£_0 - £_{45}\right)^2 + \left(£_{45} - £_{90}\right)^2}$

$£_{1,2} = \frac{\left(-220\times 10^{-6} + 120\times 10^{-6}\right)}{2}$

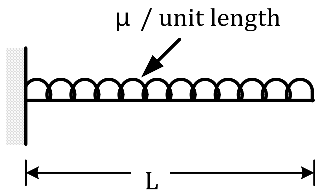
$\pm \frac{1}{\sqrt{2}}\sqrt{\left(1.156\times 10^{-7}\right) + 1\times 10^{-8}}$

$£_{1,2} = -5\times 10^{-5} \pm 2.5059\times 10^{-4}$

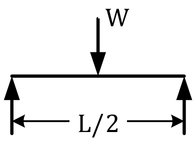
$£_1 = 2.0059\times 10^{-4}$

$£_2 = -3.0059\times 10^{-4}$

34. (B)



$M_{max} = \frac{\mu L^2}{2}$



$$M_{\max} = \frac{W\left(\frac{L}{2}\right)}{4} = \frac{WL}{8}$$

$$\therefore \quad \frac{\mu L^2}{2} = \frac{WL}{8}$$

$$\boxed{W = 4\mu L}$$

35. (15 to 18)

Gravity force and spring force are both conservative forces so the principle conservation of energy can be applied.

$m = 5kg, k = 500\,N/m$

Position 1. $P.E. = mg(0.15) + 0$

(mass) (spring)

$K.E. = 0$

$E_1 = mg(0.15) = 5 \times 9.81 \times 0.15$

Position 2. Length of the spring in position 2.

$CB = \sqrt{20^2 + 15^2} = 25\,cm$

$P.E. = 0 + \frac{1}{2}kx^2$ x is extension of spring

$(mass) \quad (spring)$

$$P.E. = \frac{1}{2}k(0.25 - 0.20)^2 = \frac{1}{2}k(.05)^2$$

$$K.E. = \frac{1}{2}mv^2 + 0$$

$(mass) \quad (spring)$

Total Energy, $E_2 = \frac{1}{2}k(0.05)^2 + \frac{1}{2}mv^2$

$$E_2 = \frac{1}{2} \times 500(0.05)^2 + \frac{1}{2} \times 5v^2$$

$E_1 = E_2$

$$5 \times 9.81 \times 0.15 = \frac{1}{2} \times 500(0.05)^2 + \frac{1}{2} \times 5v^2$$

$7.36 = 0.625 + 2.5v^2$

$v = 16.4\,m/s$

36. (0.095 – 0.110)

Mass of the bullet $m = \frac{25}{1000} = 0.025\,kg$

Velocity of the bullet $= v$

Mass of the canoe $= 35\,kg$

Mass of the man $= 70\,kg$

Velocity of the canoe $= V$

Velocity of the block $= 5m/s$

Consider the bullet and the wooden block as a system.

Initial momentum of the bullet = Final momentum of the bullet and the block.

$v(0.025) = 5(2.25 + 0.025)$

$$v = \frac{5 \times 2.275}{0.025}$$

Velocity of the bullet $v = 455\,m/s$

Next consider the bullet and the canoe with man.

Momentum of the bullet = Momentum of the canoe and the man.

$0.025v = (70 + 35)V$

Substituting for the velocity of the bullet

$v = 455\,m/s$

$$V = \frac{0.025 \times 455}{35}$$

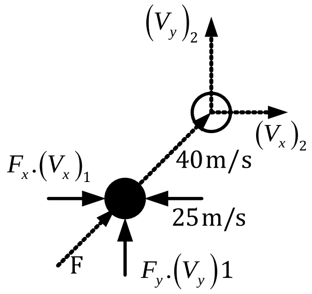
Velocity of the canoe $V = 0.108\,m/s$

37. (*)

Let us apply the principle of impulse and momentum to the ball in the x and y directions.

The component equation in the x-direction,

$$(mv_x)_2 - (mv_x)_1 = \int_0^t F_x\,dt$$



Substituting,

$$m = \frac{100}{1000} = 0.1\,kg, \Delta t = 0.015\,s$$

$(v_x)_1 = -25\,m/s$

$(v_x)_2 = 40\cos 40^0 = 30.64\,m/s$

$$\int_0^t F_x\,dt = (F_x)_{\text{average}}(\Delta t)$$

$(F_x)_{\text{average}}(\Delta t) = 0.1(30.64) - 0.1(-25)$

$$(F_x)_{\text{average}} = \frac{5.564}{0.015}$$

$(F_x)_{\text{average}} = 370.9\,N$

The equation in the y-direction.

$$(mv_y)_2 - (mv_y)_1 = \int_0^t F_y\,dt$$

Substituting, $(v_y)_1 = 0, \Delta t = 0.015\,s$

$(v_y)_2 = 40\sin 40^0 = 25.72\,m/s$

$\int_0^t F_y dt = (F_y)_{\text{average}} \times (\Delta t)$

$(F_y)_{\text{average}} \times (0.015) = 0.1(25.72) - 0.1(0)$

$(F_y) = \dfrac{2.572}{0.015}$

$(F_y)_{\text{average}} = 171.5\,N$

$F_{\text{average}} = \sqrt{(F_x)^2_{\text{average}} + (F_y)^2_{\text{average}}}$

$= \sqrt{(370.9)^2 + (171.5)^2}$

$F_{\text{average}} = 408.6\,N$

38. (6.60 to 6.65)

Initial velocity of the train,

$u = 0$

Final velocity of the train,

$v = 90\,km/hour = 25\,m/s$

Time taken $t = 50\,s$

Using $v = u + at$

Acceleration $a = \dfrac{25}{50} = 0.5\,m/s^2$

The force required to accelerate the train = Mass × acceleration of the train

$= 500 \times 10^3 \times 0.5 = 250 \times 10^3\,N$

Frictional force present $= 15 \times 10^3\,N$

Total force required = Force required to over come friction + Force required to accelerate the train

The maximum power is required at

$t = 50\,s$

When $v = 25\,m/s$

Power = $F.v = (250+15)10^3 \times 25W$

Power $= 6.625\,MW$

At any time after 50 seconds, the force is required only to overcome the frictional resistance of $15 \times 10^3\,N$.

Power required

Power required = $Fv = 15 \times 10^3 \times 25\,watt$

Power $= 375\,kW$

39. (A)

The equation of parabolic load

$x^2 = KW$

At $x = 4,\ w = 1500$

$$\boxed{K = \frac{16}{1500}}$$

Resultant of distributed load $W = \int_0^L wdx$

$= \int_0^L \dfrac{x^2}{k} dx$

$= \dfrac{1500}{16} \times \left[\dfrac{4^3}{3}\right]$

$= 2000\,N$

Line of action $(\bar{x})$

$2000 \times \bar{x} = \int_0^L (w\,dx)x$

$= \int_0^L \dfrac{x^2}{K} dx.x$

$2000 \times \bar{x} = \dfrac{1500}{16}\left[\dfrac{4^4}{4}\right]$

$$\boxed{\bar{x} = 3m}$$

40. ($a^4/5$)

$I_x = \int y^2 dA$

$= \int_0^a y^2.(x\,dy) = \int_0^a y^2.\dfrac{y^2}{a}dy = \dfrac{a^4}{5}$

41. (4400 to 4600)

$J_C = J_A - Ad^2$

$= 22.5 \times 10^2 - 6^2 \times 50$

$= 22.5 \times 10^2 - 1800$

$= 4.5 \times 10^2$

$\bar{I}_x + \bar{I}_y = \bar{J}_c$

$\Rightarrow\ 3\bar{I}_y = \bar{J}_c$

$\Rightarrow\ \bar{I}_y = 1.5 \times 10^2\,cm^4,\ \bar{I}_x = 3 \times 10^2\,cm^4$

$\bar{I}_x + \bar{I}_y = 4.5 \times 10^2\,cm^4$

42. (A)

43. $[49 \times 10^4$ to $51 \times 10^4]$

Work of spring = K.E. of car

$= \left(\dfrac{\text{Resistance of spring in kg/cm}}{2}\right)$

$\times (\text{Deflection of spring})^2$

or $\quad \dfrac{1}{2}\dfrac{W}{g}v^2 = \dfrac{R}{2} \times h^2$

or $\quad \dfrac{50,000}{9.81} \times (100)^2 = R \times 10^2$

or $\quad R = \dfrac{500 \times 10^4}{9.81} \cong 50 \times 10^4\,kg/cm$

44. (0.30 to 0.35)

When the disc rolls down an inclined plane, apart from possessing translator motion, it rotates about an axis passing through its centre of gravity and perpendicular to its plane.

$\therefore$ K.E. of rotation of the disc about this axis

$$= \frac{1}{2} I \mu^2 = \frac{1}{2}\left(\frac{1}{2} Mr^2\right)\mu^2 = \frac{1}{4} Mr^2 \mu^2 = \frac{1}{4} Mv^2$$

Here, I = M.I. of the disc about the said axis,

M = mass of the disc,

r = radius of the disc

v = linear velocity of the disc,

μ = angular velocity of the disc.

KE. Of translation $= \frac{1}{2} Mv^2$

Total energy of the disc $= \frac{1}{4} Mv^2 + \frac{1}{2} Mv^2 = \frac{3}{4} Mv^2$

$\therefore$ fraction of total energy associated with rotation

$$= \frac{\frac{1}{2} Mv^2}{\frac{3}{4} Mv^2} = \frac{1}{3}$$

45. (B)

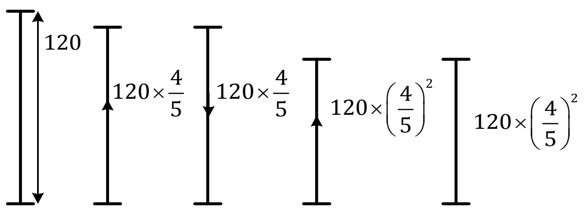
By perpendicular axis theorem

$I_z = I_x + I_y$

$= 0.2 + 0.2$

$= 0.4\, kg - m^2$

46. (1079 to 1081)



Total distance

$$= 120 + 2 \times 120 \times \frac{4}{5} + 2 \times 120 \times \left(\frac{4}{5}\right)^2 + ... + \infty$$

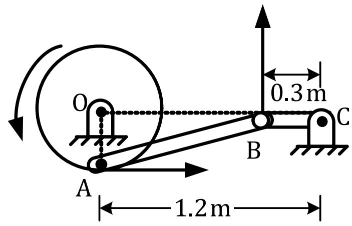
$$= 2 \times 120 + 2 \times 120 \times \left(\frac{4}{5}\right) + 2 \times 120 \times \left(\frac{4}{5}\right)^2 .... - 120$$

$$= 2 \times 120 \left(1 + \frac{4}{5} \times \left(\frac{4}{5}\right)^2 + ... + \infty\right) - 120$$

$$= 2 \times 120 \times \frac{1}{1 - \frac{4}{5}} - 120$$

$= 1200 - 120 = 1080\, m$

47. (1.75 to 1.85)



$V_A = 2 \times 0.3 = 0.6\, m/\sec$

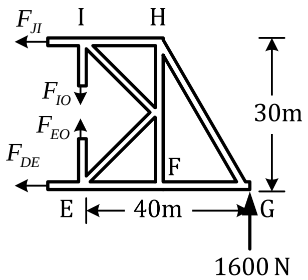
$$\frac{V_A}{0.3} = \frac{V_B}{0.9}$$

$\Rightarrow \quad V_B = 0.6 \times 3 = 1.8\, m/\sec$

$\boxed{V_B = 1.8\, m/\sec}$

48. (A)

First of all find out the reactions



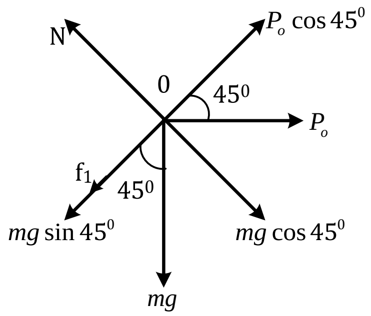
Taking moments about E,

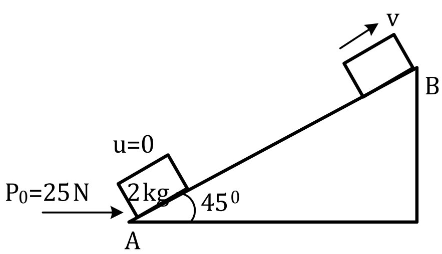
$\Sigma M_E = 0 \Rightarrow 1600 \times 40 + F_{JI} \times 30 = 0 \Rightarrow \quad F_{JI} = -2130\, N$

$\Rightarrow \quad \Sigma F_x = 0$

$\Rightarrow \quad F_{DE} = -F_{JI} = 2130\, N$

49. (1.67 to 1.69)





During motion

$W_{net} = -«mg\cos 45^0 \times 4 - mg\sin 45^0 \times 4 + P_c \cos 45^0 \times 4$

$$= 0.2 \times 2 \times \frac{10}{\sqrt{2}} \times 4 - 2 \times \frac{10}{\sqrt{2}} \times 4 + 25 \times \frac{1}{\sqrt{2}} \times 4$$

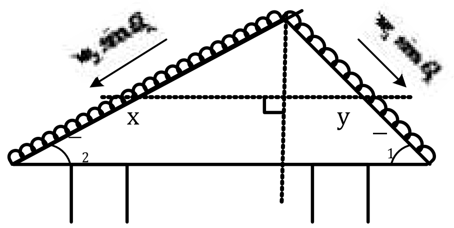
$\Rightarrow \quad 50\sqrt{2}-48\sqrt{2}=2\sqrt{2}$

$but W_{net}=\frac{1}{2}m\left(v_B^2-v_A^2\right)=\frac{1}{2}mv_B^2=2\sqrt{2}$

$v_B^2=\frac{4\sqrt{2}}{2}$

$\boxed{v_B=1.68\,m/s}$

50. (C)



$w={}^{\circ}Ax\times g$

$\sin{}^{-}{}_{2}=\frac{L}{x}$

$x=\frac{L}{\sin{}^{-}{}_{2}}$

$\sin{}^{-}{}_{2}\times w_2=\left({}^{\circ}Ag\right)\times\frac{L}{\sin{}^{-}{}_{2}}\times\sin{}^{-}{}_{2}$

Now

$\frac{w_1}{\sin{}^{-}{}_{1}}={}^{\circ}Ag\times\frac{L}{\sin{}^{-}{}_{1}}\times\sin{}^{-}{}_{1}$

$w_1\sin{}^{-}{}_{1}={}^{\circ}Ag\times L$

Hence

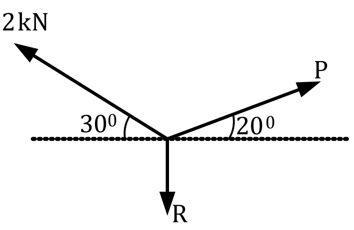
$\boxed{w_2\sin{}^{-}{}_{2}=w_1\sin{}^{-}{}_{1}}$

No movement of chain

51. (B)

Assuming resultant R to be vertical

By la of sines,



$\frac{P}{\sin 120}=\frac{2}{\sin 110}=\frac{R}{\sin 130}$

$P=1.84kN$

52. (3)

The length of the cored connecting the blocks A and B can be expressed in terms of distances x_A and x_B of the blocks A and B from a fixed point O as,

$3x_B+2x_A=$ constant ...(i)

Giving an increment Δx_A to the block A

$3\Delta x_B+2\Delta x_A=0$ ...(ii)

Differentiating (i)

$3\frac{dx_B}{dt}+2\frac{dx_A}{dt}=0$

$3v_B+2v_A=0$ ...(iii)

Differentiating again

$3\frac{d^2x_B}{dt^2}+2\frac{d^2x_A}{dt^2}=0$

$3a_B+3a_A=0$ ...(iv)

For block B, using the equation

$v^2-u^2=2as$

$u=0, v=12\,cm/s, s=24\,cm$

$a_B=\frac{(12)^2}{2\times 24}=3\,cm/s^2$

53. (6 m/s)

For perfectly elastic ball $e=1$

$m_a=2kg, m_b=6kg, m_c=12\,kg$

Impact of balls A and B

Conservation of momentum gives

$m_av_a+m_bv_b=m_av'_a+m_bv'_b$

$2\times 12+6\times 4=2v'_a+6v'_b$

$2v'_a+6v'_b=48$ ...(i)

And $e=-\frac{v'_b-v'_a}{v_b-v_a}$

$v'_b-v'_a=e\left(v_b-v_a\right)$

$v'_b=v'_a=-1(4-12)=8$ ...(ii)

Solving (i) and (ii) simultaneously

$v'_a=0$

That is the ball of mass 2 kg is brought to rest.

And $v'_b=8m/s$

Impact of Balls B and C

Consider now the impact of the ball B of mass 6 kg moving with the initial velocity of 8m/s with the ball C of mass 12 kg moving with the velocity of 2m/s.

Conservation of momentum gives

$m_bv_b+m_cv_c=m_bv'_b+m_cv'_c$

$6\times 8+12\times 2=6v'_b+12v'_c$

$v'_b+2v'_c=12$ ...(iii)

Also, $e=-\frac{\left(v'_c-v'_b\right)}{\left(v_c-v_b\right)}$

$v'_c-v'_b=e\left(v_c-v_b\right)=-1(2-8)$

Or $v'_c - v'_b = 6$

Solving (iii) and (ii) simultaneously

$v'_b = 0$, Final velocity of the ball B is zero or it is brought to rest after the second impact. And $v'_c = 6\,m/s$

54. (0.10 to 0.11)

$mgh = \frac{1}{2}mv_a^2$

Or $v_a = \sqrt{2gh}$

Let, the velocity of the ball A after impact $= v'_a$

The initial velocity of the ball $B = v_b = 0$

The velocity of the ball B after impact $= v'_b$

The ball B after the impact should attain a velocity v'_b just sufficient to rise to a height of 20 cm.

Applying the principle of conservation of energy to the motion of the ball B between position 2 and 3,

$$\frac{1}{2}\left(\frac{m}{4}\right)(v'_b)^2 = \frac{m}{4}(g)(0.2)$$

$(K.E.)$ $\quad$ $(P.E.)$

The velocity of the ball B after impact should be

$v'_b = \sqrt{2g \times 0.2}$

$= \sqrt{0.4g}$

Applying the principle conservation of momentum to the impact of balls A and B

$m_a v_a + m_b v_b = m_a v'_a + m_b v'_b$

$m\left(\sqrt{2gh}\right) + 0 = mv'_a + \frac{m}{4}\left(\sqrt{0.4g}\right)$

$-v'_a + \sqrt{2gh} = 0.945$ ...(i)

The coefficient of restitution relation gives

$e = -\frac{v'_b - v'_a}{v_b - v_a}$

$e(v_b - v_a) = -v'_b + v'_a$

$0.8\left(0 - \sqrt{2gh}\right) = -\sqrt{0.4g} + v'_a$

$v'_a = 0.8\sqrt{2gh} = 1.981$

Solving (i) and (ii)

$h = 0.104\,m$ or $h = 10.4\,cm$

55. (4)

Weight × Distance of free fall + Weight × Displacement of spring (h) = Work of spring

=Average force of spring × h^2

$1000 \times 8 + 1000 \times h = \frac{500}{2} \times h^2$

Or $32 + 4h = h^2$

Or $h^2 - 4h - 32 = 0$

Or $(h-8)(h+4) = 0$

or $h = 8cm$

or $= -4\,cm$

Taking only magnitude

$= 4cm$

56. (A)

57. (B)

58. (B)

A synagogue is a place of worship while the other three are all places related to death

59. (B)

60. (D)

A dart is a small type of spear while a gun is a small type of canon

61. (D)

Let N be the number of days Nikhil takes to complete the job working alone and A be the number of days Ajnkit takes to finish the job working alone. Thus we have, 1/N = 1/24. Now, if Nikhil worked twice as efficiently, he will take N/2 days to complete the job alone and if Ankit works 1/3 rd as efficiently, he will take 3A days to finish the same job alone , Thus we can say that 1/(N/2) + 1/3A = 1/18 Solving we get 2/3N = 1/18 – 1/72 => 5/3N = 3/72 = > N = 40 days

Alternate Method:

LCM of 24 and 18 is 72. So let us assume the total work to be 72 units If rate of working for Nikhil is n units /day and rate of working for Ankit is a units/day, we can say that n + a = 72/24 = 3 (Total rate of working = Total work/Total time taken).

Also, 2n + a/3 = 72/28 = 4. Solving both equations, we get n = 9/5 units/day. Thus the time taken by Nikhil to finish the job alone = 72/(9/5) = 40 days,

62. (D)

Sum of n terms = $n^2 + 3n$. Sum of (n-1) terms = $(n-1)^2 + 3(n-1) = n^2 + n - 2$

Now, we know, n^{th} term = Sum of n terms – Sum of (n-1) terms, Therefore, n^{th} in this case = $n^2 + 3n - (n^2 + n - 2) = 2n + 2$. Therefore, 5^{th} terms is 2x5 + 2 = 12

63. (C)

64. (D)

The net movement of the monkey = 2 steps in 4 seconds ie 1 step for every 2 second In our bid to solve question quickly, we may tend to directly multiply 21 steps by 2 seconds and arrive at 42 seconds. But consider this – after the monkey reaches the 18^{th} step, he has to climb another 3 steps to reach 21 which he will do in the next 3 second (the fact that he slips another step after reaching the 21^{st} is of no concern to us). Therefore total time taken by the monkey is 18 x 2 + 3 = 39 seconds.

65. **(D)**

Since we do not know that total number of students graduating from all the IITs put together, we cannot find the percentage of students who did not get placed and hence the data is insufficient