## ANSWER KEY

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | 13.33 N/m² | **13.** | C | **25.** | A | **37.** | C | **49.** | A | **61.** | D |
| **2.** | $\frac{m\omega^2 L}{2A}$ | **14.** | D | **26.** | C | **38.** | D | **50.** | A | **62.** | D |
| **3.** | B | **15.** | A | **27.** | 3m | **39.** | B | **51.** | D | **63.** | C |
| **4.** | A | **16.** | B | **28.** | $\frac{2WL}{9\sqrt{3}}$ | **40.** | C | **52.** | A | **64.** | D |
| **5.** | C | **17.** | C | **29.** | 0.5359m | **41.** | D | **53.** | C | **65.** | D |
| **6.** | D | **18.** | 40% | **30.** | 31 – 32 | **42.** | $\frac{R+a}{R}\sqrt{\frac{4k}{3M}}$ | **54.** | B | | |
| **7.** | D | **19.** | 0.742 | **31.** | 252.6 N/m² | **43.** | 11.75 kgm² | **55.** | A | | |
| **8.** | 1.93 m | **20.** | B | **32.** | 9.417 kPa | **44.** | A | **56.** | A | | |
| **9.** | D | **21.** | C | **33.** | 4708.8 Pa | **45.** | C | **57.** | B | | |
| **10.** | 0 | **22.** | C | **34.** | 30.19 Pa | **46.** | B | **58.** | B | | |
| **11.** | B | **23.** | A | **35.** | 784.794 kPa | **47.** | C | **59.** | B | | |
| **12.** | D | **24.** | D | **36.** | B | **48.** | A | **60.** | D | | |

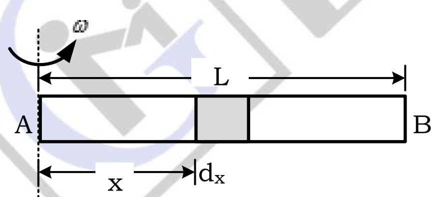
## SOLUTIONS

**1. (13.3 – 13.4)**

$$\tau_{avg} = \frac{P}{A} = \frac{100}{10} = 10\,N/m^2$$

Then, $\tau_{\max} = \frac{4}{3} \times \tau_{avg} = \frac{4}{3} \times 10 = \frac{40}{3} = 13.33\,N/m^2$

**2.** $\left(\frac{m\omega^2 L}{2A}\right)$




Consider an element of length $dx$ at a distance $x$ from end $A$.

So, $dF = (dm) \times x \times \omega^2$

Now, $dm = \frac{m}{L} dx$

So, $dF = \frac{mx\omega^2 dx}{L}$

Maximum force will be at a cross section about which it is rotating

$$F_{\max} = \int_0^L \frac{mx\omega^2 dx}{L}$$

$$\therefore \quad F_{\max} = \frac{m\omega^2 L}{2}$$

Maximum stress will be $\sigma = \frac{F_{\max}}{A} = \frac{m\omega^2 L}{2A}$

**3. (B)**

The density of fluid,

$\rho = S.G \times 1000\ kg/m^3$

$= 850\ kg/m^3$

Pressure at depth h from free surface

$= \rho g h$

Pressure at inside tank,

$= P_{atm} + \rho g h$

$$P = 96\ kPa + \frac{850 \times 9.81}{1000} \times 0.55\ kPa$$

$$P = 100.586\ kPa$$

**4. (A)**
For free falling body relative acceleration due to gravity is zero
$\therefore \quad P=\rho gh$ if $g=0$ then $P=0$
(but it is only hydrostatic pressure) and these will be atm. Pressure throughout the liquid.

**5. (C)**
Absolute pressure = Atm pressure + gauge pressure
$=(25+0.13)\text{bar}=26.03 \text{ bar}$

**6. (D)**
Since there is no flow of water in pipe.
So there no raise or fall of meniscus in tube.
Mercury will balance.

**7. (D)**
Tension in string and weight of body is equation to the upward buoyancy force.
$T+Mg=\text{Buoyancy force}$
$T+\rho_S HAg=\rho hAg$
$T=gA(\rho h-\rho_S H)$

**8. (1.93 m from bottom surface.)**
Apply the hydrostatic formula from point A to point B.
$P_A+(\rho gh)_{\text{air}}+(\rho gh)_{\text{gasoline}}+\{\rho g(h-z)\}_{\text{glycerin}}=P_B$

$P_A=1500\ Pa,\ P_B=0$
$1500+12.0\times 2.0+6670\times 1.5+12360(1-z)=0$
$1500+24+10005+12360(1-z)=0$
Solving for z, $\quad z=1.93m$

**9. (D)**
4 = 3x and v = -3y
The equation of stream line in two Dimensional flow is
$\frac{dy}{dx}=\frac{v}{u}\Rightarrow\frac{dx}{u}=\frac{dy}{v}$
On integration
$\int\frac{dx}{3x}=\int-\frac{dy}{3y}$
$\frac{1}{3}\left|nx=-\frac{1}{3}\right|ny+\frac{1}{3}\left|nc\right.$ where c – constant
$|nxy|nc$ or $ny=c$
For stream line passing through 1,1) , c = 1
The required stream line equation is xy = 1

**10. (0)**
From the definition stream function $u=\frac{\partial\psi}{\partial y}$ and
$v=\frac{-\partial\psi}{\partial x}$, as $\psi$=xy
$v=\frac{-\partial\psi}{\partial x}=-y, u=\frac{\partial\psi}{\partial y}=x$
$u=x, v=-y$
For irrotational flow $\frac{\partial v}{\partial x}=\frac{\partial u}{\partial y}, \frac{\partial u}{\partial y}=0, \frac{\partial v}{\partial x}=0, \therefore$ flow is irrotational for irrotational flow vorticity is zero (0)

**11. (B)**
A valid potential function must satisfy the laplace equation
(i) $\quad \phi=2x+5y$
$\frac{\partial\phi}{\partial x}=2$ and $\frac{\partial\phi}{\partial y}=5$
$\frac{\partial^2\phi}{\partial x^2}=0$ and $\frac{\partial^2\phi}{\partial y}=0$
Hence $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$
$\phi=2x-5y^2$ is a valid potential function
(ii) $\quad \phi=4x^2-5y^2$
$\frac{\partial\phi}{\partial x}=8x$ and $\frac{\partial\phi}{\partial y}=-10y$
$\frac{\partial^2\phi}{\partial x^2}=8$ and $\frac{\partial^2\phi}{\partial y}=-10$
Hence $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=-2\neq 0$
Hence $\phi=4x^2-5y^2-5y^2$ is not a valid potential function.

**12. (D)**
For irritation flow following condition must be satisfied
$\omega_z=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)=0$
(i) $u=3xy; v=\frac{3}{2}x^2-\frac{3}{2}y^2$
$\omega_2=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)=\frac{1}{2}(3x-3x)=0$
Hence flow is irrotational
(ii) $u=y^2, v=3x$
$\omega_z=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)=\frac{1}{2}(-3-2y)\neq 0$
Hence flow is not irrotational.

**13. (C)**

**14. (D)**

**15. (A)**

**16. (B)**

$$A_1V_1 = A_2V_2$$
$$d_1^2V_1 = d_2^2V_2$$
$$\Rightarrow \quad V_2 = \frac{20^2}{10^2}\times 4 = 16m/s$$

Velocity head at sec 2 $= \frac{V_2^2}{2g} = \frac{16^2}{2\times 9.81} = 13m$

**17. (C)**

$(head)_{total}$ at section 1 $= \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1$

$$= \frac{29.4\times 10^4}{10^3\times 9.81} + \frac{25^2}{2\times 9.81} + 28$$

$(head)_{total}$ at section 2 $= \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$

$$= \frac{22.5\times 10^4}{10^3\times 9.81} + \frac{25^2}{2\times 981} + 30$$

Difference between the two sections = 5.0 m

**18. (40)**

**Given:**

$$\frac{\omega_{\max}}{\omega_{\min}} = 1.5$$

Now, $C_f = \dfrac{\omega_{\max}-\omega_{\min}}{\dfrac{\omega_{\max}+\omega_{\min}}{2}} = \dfrac{2(\omega_{\max}-\omega_{\min})}{(\omega_{\max}+\omega_{\min})}$

$$= \frac{2(1.5-1)}{(1.5+1)} = \frac{2\times 0.5}{2.5} = \frac{2}{5} = 40\%$$

**19. (0.742)**

**Case 1:**

$$\omega_1 = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{10}}$$

**Now, Case 2:**

$$\omega_2 = \sqrt{\frac{2k}{m+\frac{m_g}{3}}} = \sqrt{\frac{2k}{10+\frac{3}{3}}} = \sqrt{\frac{2k}{11}}$$

**20. (B)**

**Given:**

$$P = 250\,kW, N_1 = 1800\,rpm, F = 22.1048\,kN, T_1 = 14$$

So, $P = T\omega_1 = T\times\left(\frac{2\pi N_1}{60}\right) = T\times\left(\frac{2\pi\times 1800}{60}\right)$

$$\Rightarrow \quad T = \frac{250\times 60}{2\pi\times 1800} = 1.326\,kNm$$

Also, $T = \frac{F\times D_1}{2}$

$$\therefore \quad D_1 = \frac{2T}{F} = \frac{2\times 1.326}{22.1048} = 0.11997\,m = 119.974\,mm$$

Now, $\frac{D_1}{2} + \frac{D_2}{2} = 660$

$$\Rightarrow \quad D_2 = 1200.026\,mm$$

Also, by equal module of gear 1 and 2,

$$\frac{D_1}{T_1} = \frac{D_2}{T_2}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{D_2}{D_1} = \frac{1200.026}{119.974} = 10$$

**21. (C)**

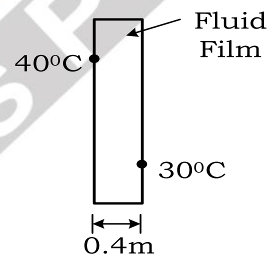
**Given:** $C = 40\,N/m/s, C_c = 420\,N/m/s$

So, $\xi = \frac{C}{C_c} = \frac{40}{420} = 0.095$

Now, logarithmic decrement $(\delta)$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi\times 0.095}{\sqrt{1-0.095^2}} = 0.6$$

**22. (C)**




From Fourier's law of heat conduction

$$Q = -KA\frac{dT}{dx} = \frac{0.14(40-30)}{0.4\times 10^3}$$

$$Q = 3500\,W/m^2$$

**23. (A)**

$$UA = \frac{1}{\Sigma R} = \frac{1}{\frac{1}{h}+\frac{b}{k}+\frac{1}{h}}$$

$$UA = \frac{1}{\frac{1}{10}+\frac{1}{20}+\frac{5\times 10^{-2}}{1}}$$

$$\frac{1}{UA} = \frac{1}{10}+\frac{1}{20}+\frac{1}{50}$$

$$= \frac{10+5+2}{100} = \frac{17}{100}$$

$$UA = \frac{100}{17} = 5.88$$

**24. (D)**

$$Q = \frac{(T_1-T_2)}{R_{th}}$$

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$$= \frac{(1840-340)}{\frac{0.3}{0.6\times1}+\frac{0.2}{0.4\times1}+\frac{0.1}{0.1\times1}}$$

$$= 750W$$

**25. (A)**

**26. (C)**

$$y_{max} = \frac{1}{3}m$$

$$\sigma_t = \frac{3\times10^6\times1000\times\frac{1}{3}\times1000}{\frac{1}{36}\times1000^4} = 36\,MPa$$

**27. (3)**

$\Sigma V = 0$

$\Rightarrow \quad R_A + R_B = 2 + 2\times4 = 10\,kN$

$\Sigma M = 0,$

$\Rightarrow \quad 2\times4\times2 + 2\times6 = R_B \times 4$

So, $R_B = \frac{16+12}{4} = \frac{28}{4} = 7\,kN$

And $R_A = 3\,kN$

Bending moment at a distance $'x'$ from end $A$ will be

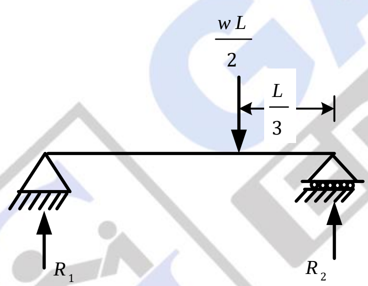
$$(BM)_x = 3x - 2\times x\times\frac{x}{2} = 3x - x^2$$

Now, the BMD changes sign in section AB, so the point of contraflexure is where the BM is zero.

So, $3x - x^2 = 0$

$x = 3m$

**28. (D)**



$\Sigma V = 0 \quad \Rightarrow \quad R_1 + R_2 = \frac{wL}{2}$

$\Sigma M = 0 \quad \Rightarrow \quad \frac{wL}{2}\times\frac{2L}{3} = R_2 \times L$

So, $R_2 = \frac{wL}{3}$

$R_1 = \frac{wL}{2} - \frac{wL}{3} = \frac{wL}{6}$

Bending moment at $x$ distance from left end,

$$BM_x = \frac{wLx}{6} - \frac{wx^2}{2L}\times\frac{x}{3} = \frac{wLx}{6} - \frac{wx^2}{6L}$$

For maximum BM, $\frac{dBM_x}{dx} = 0$

So, $\frac{wL}{6} - \frac{3wx^2}{6L} = 0$

$\Rightarrow \quad x^2 = \frac{wL}{6}\times\frac{6L}{3w} = \frac{L^2}{3}$

$\Rightarrow \quad x = \frac{L}{\sqrt{3}}$

So, from equation (i),

$$(BM)_{max} = \frac{wL}{6}\times\frac{L}{\sqrt{3}} - \frac{w}{6L}\times\frac{L^3}{\sqrt{3}\times\sqrt{3}\times\sqrt{3}}$$

$$= \frac{wL^2}{6\sqrt{3}} - \frac{wL^2}{18\sqrt{3}} = \frac{2wL^2}{18\sqrt{3}} = \frac{wL^2}{9\sqrt{3}}$$

Now, $\frac{1}{2}\times w\times L = W$

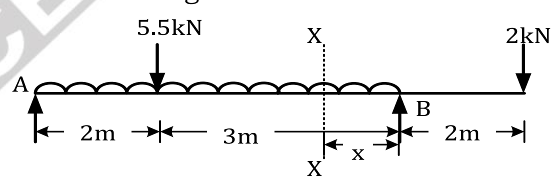
$\Rightarrow \quad wL = 2W$

Substituting in (ii),

So, $(BM)_{max} = \frac{2WL}{9\sqrt{3}}$

**29. Marks to All (0.51 – 0.56)**

Point of contraflexure is the position where bending moment changes sign. The point itself have zero bending moment



Taking moment about A

$\Rightarrow \quad R_B \times 5 = 5.5\times2 + 2\times5\times\frac{5}{2} + 2\times7$

$\Rightarrow \quad R_B = 10\,kN$

Bending moment at a section XX at a distance x from end $B$.

$\Rightarrow \quad M_x = -2[2+x] - \frac{2x^2}{2} + 10x$

$\Rightarrow \quad = -4 - 2x - x^2 + 10x$

$\Rightarrow \quad M_x = -4 + 8x - x^2$

Taking $M_x = 0,$

$\Rightarrow \quad x^2 - 8x + 4 = 0$ on solving

$\Rightarrow \quad x = 0.5359\,m$ as other value of $x$ is not admissible.

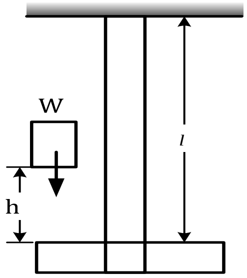
**30. (31-32)**

$$\sigma_1 = \frac{P}{A} = 31.831\,kPa$$

$$\sigma_2 = \frac{M}{Z} = 0$$

So, $(\sigma_t)_{\max} = \sigma_1 + \sigma_2 = 31.83 kPa$

**31. (250 – 254)**




**Given,**

$W = 100\,N, d_{bar} = 0.02\,m, h = 5\,m, E = 200\,GN/m^2,$
$l = 10\,m$

From the relation,

$$\sigma = \frac{W}{A}\left[1+\sqrt{1+\frac{2hAE}{WL}}\right]$$

$$\Rightarrow \quad \sigma = \frac{100}{\frac{\pi}{4}\times(0.02)^2}\left[1+\sqrt{1+\frac{2\times5\times\frac{\pi}{4}(0.02)^2\times200\times10^9}{100\times10}}\right]$$

$$= 252.6\times10^6\,N/m^2$$

$$\sigma = 252.6\,MN/m^2$$

**32. (9.417 kPa)**

Equating the pressure on both the limbs at horizontal plane xx

$P_A + [0.08+0.13+0.08]\times\rho_A\times g$

$= P_B + 0.13\times\rho_B\times g + 0.08\times\rho_m\times g$

$P_A + [0.29\times800\times9.81] = P_B + 0.13\times800\times9.81$

$+0.08\times13600\times9.81$

$P_A - P_B = 1020.24 + 10673.28 - 2275.92$

$P_A - P_B = 9417.6\,Pa$

$P_A - P_B = 9.417\ kPa$

**33. (4708.8 Pa)**

Take reference line passing through point A
Pressure will same on reference line,
Pressure at, $A = \rho gh$

$P_A = 0.8\times1000\times9.81\times8\times10^{-2}\,pa$

$P_A = 627.84\ Pa$

And pressure at, $B = \rho g\left(\frac{8}{100}-\frac{60}{100}\right)Pa$

$= 80\times9.81\times\frac{-52}{100} = -4080.96\ Pa$

Difference in pressure,

$P_A - P_B = 627.84 - (-4080.96)$

$= 4708.8\ Pa$

**34. (30.19 Kpa)**

Reassure at B, $P_B = P_A - [0.9\times1000\times9.81\times1.2]$

$P_B = 4000 - [900\times9.81\times1.2]$

$P_B = 29.4052\ Pa$

$P_C = P_B - [0.9\times1000\times9.81\times0.8] = 22342\ Pa$

$P_C = P_D = 22342\,Pa$

$P_E = P_D + [1000\times9.81\times0.8] = 30190\ Pa$

$= 30.19\ kPa\,(\text{gauge})$

**35. ( 784.794kPa )**

According to the condition for static equilibrium, it y is measured downwards from free surface.

$\frac{-\partial P}{\partial y} + \rho g = 0$ or $dp = \rho g dy$

$dp = (1000 + 0.01y)\,g\,dy$

Integration of above equation gives,

$$P = \left(1000y + 0.01\frac{y^2}{2}\right)g + c$$

Where c is constant of integration. Since gauge pressure p at the free surface (where y = 0) is zero, $c = 0$.

Hence, $P = \left(1000y + 0.01\frac{y^2}{2}\right)g$

At $y = 80\,m$ $\quad P = \left(1000\times80 + \frac{0.01\times80^2}{2}\right)\times9.806$

$P = 784793.792\ Pa$

**36. (B)**

To check for steady flow use continuity equation .

(i) $\frac{\partial u}{\partial x} = 1, \frac{\partial v}{\partial y} = -1 \quad \therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$ So the flow is steady

(ii) $\frac{\partial u}{\partial x} = t^2, \frac{\partial v}{\partial y} = -t^2 \quad \therefore$ Statistics the continuity equation so the flow is steady

(iii) $\frac{\partial u}{\partial x} = t^2, \frac{\partial v}{\partial y} = xt + 2y$

This does not satisfy the requirement for steady flow.

To check for irrotational flow $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$

(i) $\frac{\partial u}{\partial y} = 1, \frac{\partial v}{\partial x} = 1 \quad \therefore$ Flow is irrotational

(ii) $\frac{\partial u}{\partial y} = 1, \frac{\partial v}{\partial x} = 2x \quad \therefore$ Flow is not irrotational

(iii) $\frac{\partial u}{\partial y} = 0\,\frac{\partial v}{\partial x} = yt \quad \therefore$ Flow is not irrotational

**37. (C)**

$\phi = (x^2 - y^2) + 3xy$

$u = \frac{\partial \phi}{\partial x} = 2x + 3y = \frac{\partial \psi}{\partial y}$

$\psi = 2xy + \frac{3}{2}y^2 + f(x)$ (i)

$v = \frac{\partial \phi}{\partial y} = -2y + 3x = -\frac{\partial \psi}{\partial x}$ (ii)

And from (i) $\frac{-\partial \psi}{\partial x} = -2y - f'(x)$

Thus $f'(x) = -3x$ and hence

$\psi = 2xy - \frac{3}{2}x^2$

The required stream function is

$\psi = 2xy - \frac{3}{2}x^2(x^2 - y^2)$

At point (1,1) $\psi = 2xy - \frac{3}{2}(x^2 - y^2)$

At point (1,2) $\psi_2 = \left[2 \times (1 \times 2) - \frac{3}{2}(1 - 4)\right] = 8.5 unit$

Flow rate between the stream line passing through (1,1) and (1,2)

$= \Delta\psi = \psi_2 - \psi_2 = (8.5 - 2.0) = 6.5 unit$

**38. Marks to all (360.55 unit)**

$\phi = 5(x^2 - y^2)$

$\frac{\partial \phi}{\partial x} = 10x; \frac{\partial \phi}{\partial y} = -10y$

But velocity component u and v are given by definition as

$u = \frac{-\partial \phi}{\partial x} = -10x$

$v = \frac{-\partial \phi}{\partial y} = -(-10y) = 10y$

Acceleration can be calculated by

$a_x = \frac{u\partial u}{\partial x} + \frac{v\partial u}{\partial y}$

$a_x = -10x \times (-10) + 10y \times (0) = 100x$

$a_y = \frac{u\partial y}{\partial x} + \frac{v\partial v}{\partial y}$

$a_y = -10x \times 0 + 10y \times 10 = 100y$

At (3,2) $a_x = 300$

$a_y = 200$

Acceleration $a = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{300^2 + 200^2} = 360.55$ unit

**39. (B)**

We know that

$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$ where $\phi$ - potential function

And $\frac{\partial \phi}{\partial y} = \frac{-\partial \psi}{\partial x}$ $\psi$ - stream function.

$\frac{\partial \psi}{\partial y} = \frac{\partial}{\partial x}(x(2y-1) = 2y - 1)$ ...(1)

$\frac{\partial \psi}{\partial x} = -\frac{-\partial}{\partial y}(x(2y-1)) = -(2x) = --2x$ ...(2)

Integrate equation (1) w.r.t 'y' we get

$\int d\psi = \int (2y-1)dy$ or $\psi = \frac{2y^2}{2} - y + c$

C-constant It can be function of x let the of constant K then

$\psi = y^2 - y + K$ ...(3)

Differentiating the above equation w.r.t . x we get

$\frac{\partial \psi}{\partial x} = \frac{\partial k}{\partial x}$

But $\frac{\partial \psi}{\partial x}$ from equation (2) =-2x

Then $\frac{\partial k}{\partial x} = -2x$

Integrating this equation

$K = \int -2x dx = -\frac{-2x^2}{2} = -x^2$ substitute this value of K in (3) we get $\psi = y^2 - y - x^2$

**40. (C)**

We know that

$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$ where $\phi$ - potential function

And $\frac{\partial \phi}{\partial y} = \frac{-\partial \psi}{\partial x}$ $\psi$ - stream function.

$\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial y}(2xy) = 2x$ ...(1)

$\frac{\partial \phi}{\partial y} = \frac{-\partial}{\partial x}(2xy) = -2y$ ...(2)

Integrating equation (1) we get $\int d\phi = \int 2x dx$

$\phi = \frac{2x^2}{2} + c = x^2 + c$ ...(3)

Where c is constant It can be function of y only differentiating equation (3) w.r.t. we get $\frac{\partial \phi}{\partial y} = \frac{\partial c}{\partial y}$

From equation (2) $\frac{\partial \phi}{\partial y} = -2y$

$\therefore \frac{\partial c}{\partial y} = -2y$

Integrating this equation we get

$$c = \int 2y\,dy = \frac{-2y^2}{2} = -y^2$$

Substituting this value of c in equation

$$\phi = x^2 - y^2$$

**41. (D)**

In the top plane, pressure at any radial distance r is

$$P = \frac{\rho(\omega r)^2}{2}$$

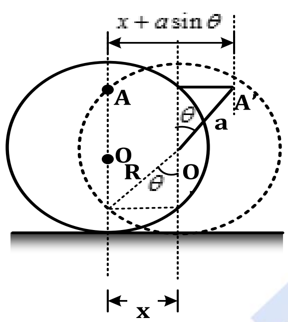
and thrust on top plane $F_T = P.\int_0^R 2\pi r dr$

$$\Rightarrow \quad F_T = \pi\rho\omega^2 \int_0^R r^3 dr = \frac{\pi\rho\omega^2 R^4}{4}$$

And, $\omega = \dfrac{2\pi \times 200}{60} = 20.94\ rad/\sec$

putting the values $F_T = 551N$

**42.** $\left(\dfrac{R+a}{R}\sqrt{\dfrac{4k}{3M}}\right)$




Displacement, $AA' = x + a\sin\theta = x + a\theta$ (For small angle, $\sin\theta = \theta$)

So, $AA' = (R+a)\theta$

The total energy of the system,

$$U = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}I\omega^2 + 2\times\frac{1}{2}ky^2$$

Where, $y = AA' = (R+a)\theta$ and $I = \dfrac{MR^2}{2}$

$$\text{So, } U = \frac{1}{2}M\dot{\theta}^2R^2 + \frac{1}{2}\times\frac{MR^2}{2}\times\dot{\theta}^2 + k(R+a)^2\theta^2$$

$$U = \frac{3}{4}MR^2\dot{\theta}^2 + k(R+a)^2\theta^2$$

Now, $\dfrac{dU}{dt} = 0$

$$\frac{3}{2}MR^2\dot{\theta}\ddot{\theta} + 2k(R+a)^2\theta\dot{\theta} = 0$$

$$\Rightarrow \quad \ddot{\theta} + \frac{4k}{3M}\frac{(R+a)^2}{R^2}\theta = 0$$

$$\therefore \quad \omega_n = \sqrt{\frac{4k}{3M}\frac{(R+a)^2}{R^2}} = \frac{R+a}{R}\sqrt{\frac{4k}{3M}}$$

**43. (11.3 – 12.3)**

Let the energy at A

So, $E_A = E$

$$E_B = E - 0.5$$

$$E_C = E - 0.5 + 1.2 = E + 0.7$$

$$E_D = E - 0.5 + 1.2 - 0.95 = E - 0.25$$

$$E_E = E - 0.5 + 1.2 - 0.95 + 1.45 = E + 1.2$$

$$E_F = E - 0.5 + 1.2 - 0.95 + 1.45 - 0.85 = E + 0.35$$

$$E_G = E - 0.5 + 1.2 - 0.95 + 1.45 - 0.85 + 0.71$$

$$= E + 1.06$$

$$E_H = E_A = E$$

So,

$$\Delta E = E_{\max} - E_{\min} = E_E - E_B = (E+1.2) - (E-0.5)$$

$$= 1.7cm^2 = 1.7\times 7000 \times \frac{30\pi}{180} = 6230.83\,Nm$$

Also, $\Delta E = I\omega^2 c$

$$\therefore \quad I = \frac{6230.83}{(94)^2\times 0.06} = 11.75\,kgm^2$$

**44. (A)**

$$T_S = T_\infty + \frac{qr_o}{2h}$$

$$= 75 + \frac{5\times10^7 \times 25\times10^{-3}}{2\times1000}$$

$$T_S = 700^0 C$$

**45. (C)**

For sphere

$$A_{gm} = \sqrt{A_1 A_2}$$

$$= \sqrt{2\times 8} = 4\,m^2$$

**46. (B)**

Heat lost by convectioin = Heat conducted by conduction

$$h\Delta T = \frac{T_1 - T_2}{\frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C}}$$

$$25\times 200 = \frac{600-20}{\frac{0.3}{20} + \frac{0.15}{K_B} + \frac{0.15}{50}}$$

$$\frac{5000}{580} = \frac{1}{0.018 + \frac{0.15}{K_B}}$$

$$0.116 = 0.018 + \frac{0.15}{K_B}$$

$$K_B = 1.53\,W/mK$$

**47. (C)**

From the Fourier's law of conduction

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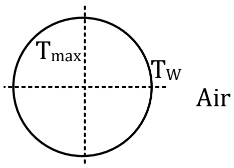
$$Q = -KA\frac{dT}{dx}$$

$$K \propto \frac{1}{dT}$$

**48. (A)**

**49. (A)**




$$T_W = \frac{\bar{q}R}{2h} + T_f$$

$$T_W - T_f = \frac{\bar{q}R}{2h}$$

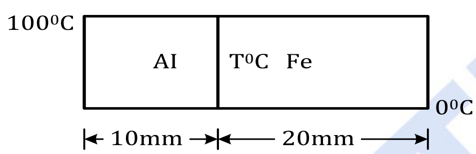
Surface heat flux $q_o = h\left(T_W - T_f\right)$

$$q_o = \frac{\bar{q}R}{2h}$$

$$\bar{q} = \frac{2q_o}{R}$$

**50. (A)**

**51. (D)** $\frac{K_{AI}}{K_{Fe}} = 3$




$$Q_{Al} = \frac{K_{Al} \times A(100 - T)}{10}$$

$$Q_{Fe} = \frac{K_{Fe} \times A(T - 0)}{20}$$

$$K_{Al}(100 - T) \times 2 = K_{Fe}(T)$$

$$3(100 - T) \times 2 = T$$

$$600 - 6T = T \quad \Rightarrow \quad 600 = \pi \Rightarrow T = \frac{600}{7} = 85.7^0 C$$

**52. (A)**

Thermal resistance $= \frac{L}{KA} = \frac{0.1}{5 \times 200} = 10^{-4} K/W$

**53. (C)**

$$Q = \frac{KA(T_1 - T_2)}{0.1}$$

$$25 \times 10^3 = \frac{50 \times 1(100 - T_2)}{0.1}$$

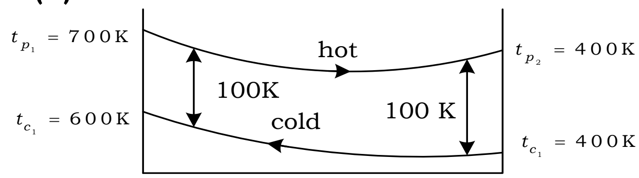
$$T_2 = 50^0 C$$

**54. (B)**

From the fourier law of heat conduction

$$K \propto \frac{1}{\Delta T}$$

**55. (A)**




**Hence LMTD = 100LK**

**56. (A)** **57. (B)** **58. (B)** **59. (B)**

**60. (D)**

A dart is a small type of spear while a gun is a small type of canon

**61. (D)**

let N be the number of days Nikhil takes to complete the job working alone and A be the number of days Ankit takes to finish the job working alone. Thus we have, 1/N = 1/24. Now, if Nikhil worked twice as efficiently, he will take N/2 days to complete the job alone and if Ankit works 1/3 rd as efficiently, he will take 3A days to finish the same job alone , Thus we can say that 1/(N/2) + 1/3A = 1/18 Solving we get 2/3N = 1/18 – 1/72 => 5/3N = 3/72 = > N = 40 days

**Alternate Method:** LCM of 24 and 18 is 72. So let us assume the total work to be 72 units If rate of working for Nikhil is n units /day and rate of working for Ankit is a units/day, we can say that n + a = 72/24 = 3 (Total rate of working = Total work/Total time taken).

Also, 2n + a/3 = 72/28 = 4. Solving both equations, we get n = 9/5 units/day. Thus the time taken by Nikhil to finish the job alone = 72/(9/5) = 40 days,

**62. (D)** Sum of n terms = $n^2$ + 3n. Sum of (n-1) terms = $(n-1)^2$ + 3 (n-1) = $n^2$ + n – 2

Now, we know, $n^{th}$ term = Sum of n terms – Sum of (n-1) terms, Therefore, $n^{th}$ in this case = $n^2$ + 3n – ($n^2$ + n – 2) = 2n + 2. Therefore, $5^{th}$ terms is2x5 + 2 = 12

**63. (C)**

**64. (D)**

The net movement of the monkey = 2 steps in 4 seconds ie 1 step for every 2 second In our bid to solve question quickly, we may tend to directly multiply 21 steps by 2 seconds and arrive at 42 seconds. But consider this – after the monkey reaches the $18^{th}$ step, he has to climb another 3 steps to reach 21 which he will do in the next 3 second (the fact that he slips another step after reaching the $21^{st}$ is of no concern to us). Therefore total time taken by the monkey is 18 x 2 + 3 = 39 seconds.

**65. (D)**

Since we do not know that total number of students graduating from all the IITs put together, we cannot find the percentage of students who did not get placed and hence the data is insufficient

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