ANSWER KEY

1. B	**2.** B	**3.** 471 kPa	**4.** 4.8 m	**5.** 50.95 m/s	**6.** 50 kg
7. 0.75	**8.** 3 Pa	**9.** 1.21 m	**10.** $\dfrac{u^2-v^2}{2g}$	**11.** B	**12.** A
13. 1.47 kN	**14.** 10.19 kN	**15.** B	**16.** 20 m	**17.** D	**18.** B
19. C	**20.** 20 m	**21.** C	**22.** B	**23.** 89%	**24.** 3.417 m/s
25. 849.52 N	**26.** D	**27.** D	**28.** 1.21	**29.** 0.316 m	

SOLUTIONS

1. [B]

Discharge,

$$q = u\times\frac{2B}{3} + 0\times\frac{B}{3} = \frac{2uB}{3}$$

Average velocity,

$$V = \frac{q}{B} = \frac{2u}{3}$$

$$\alpha = \frac{1}{V^3B}\left[\int_0^{2B/3} u^3dy + \int_{2B/3}^{B} u^3dy\right]$$

$$= \left(\frac{1}{V^3B}\right)\left(u^3\right)\left(\frac{2B}{3}\right)$$

$$= \frac{2}{3}\left(\frac{u}{V}\right)^3 = \frac{2}{3}\times\left(\frac{3}{2}\right)^3$$

$$= \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

2. [B]

Applying Bernoulli's equation between A and B

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L$$

Here, $V_A = 0,\ Z_A = Z_B$ and $h_L = 0$

$$\therefore \quad \frac{P_A}{\rho g} - \frac{V_B^2}{2g} = \frac{P_B}{\rho g}$$

or $\quad P_B = P_A - \dfrac{\rho V_B^2}{2}$

$\therefore \quad P_B < P_A$ (as $V_B \neq 0$)

3. [471 kPa]

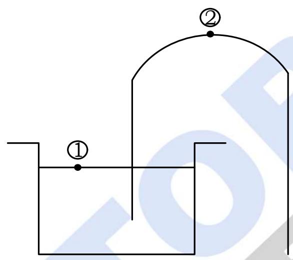
$$\frac{P_S'}{\rho g} + \frac{V_s^2}{2g} + 2 = \frac{P}{\rho g} + \frac{25^2}{2g} + 2$$

$$\Rightarrow \quad \frac{P_S}{\rho g} = \frac{1025\times9.81\times15}{1025\times9.81} + \frac{25^2}{2\times9.81}$$

P_S = 471 kPa

Assuming torpedo to be stationary & sea water assumed to move with approach velocity 25m/sec.

4. [4.8(4.7-4.9)]



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$v_2 = \frac{Q}{A} = \frac{0.02}{\frac{\pi}{4}\times0.05^2} = 10.19\,m/s$$

$v_1 = z_1 = ;\ z_2 = h$

$P_2 = 2.34\times10^3$ Pa

$P_1 = 101325$ Pa

$$\frac{101325}{10^3\times9.81} + 0 + 0 = \frac{2340}{10^3}\times9.81 + \frac{10.19^2}{2\times9.81} + h$$

$h = 10.3287 - .2385 - 5.292$

$= 4.798$ m $\simeq$ **4.8 m**

5. [50.95 m/s]

$$Q = 2.5m^3/s$$

$$d_1 = 50\,cm = 0.5\,m$$

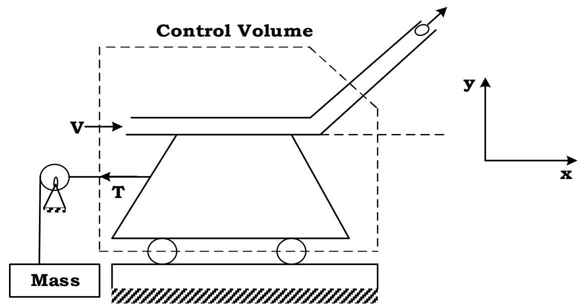
$$d_2 = 25\,cm = 0.25\,m$$

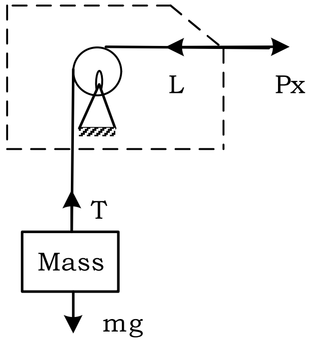
$$Q = \frac{\pi}{4}d_2^2V_2$$

$$2.5 = \frac{3.14}{4}\times(0.25)^2\times V_2$$

or $\quad V_2$ = **50.95 m/s**

6. [50 kg]





Force exerted by liquid on vane in x-direction,

F_x = Momentum Out – Momentum In

$F_x = -T = \rho Q(V_{\cos\theta} - V)$

$T = \rho AV^2(1 - \cos\theta)$

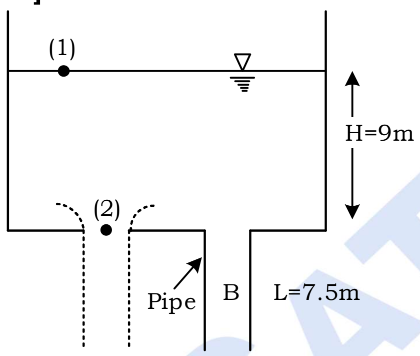
$$= 1000 \times \frac{1}{16} \times (4)^2 (1 - \cos 60°)$$

T = 500N

T = mg

$$m = \frac{500}{10} \Rightarrow 50 \text{ kg}$$

7. [0.75]



As all other parameters remain constant

$Q \propto V$

$\propto \sqrt{2gh}$

For Orifice, $Q_{Ori} \propto \sqrt{H}$

For pipe, $Q_{pipe} \propto \sqrt{H + L - loss}$

$$\frac{Q_{ori}}{Q_{pipe}} = \sqrt{\frac{9}{9 + 7.5 - 0.5}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

= 0.75

8. [3]

Given :

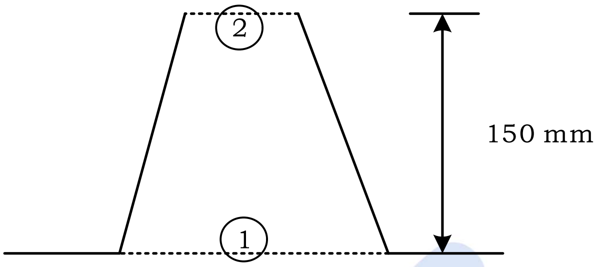
Flow can be assumed to be inviscid with constant density.

Applying Bernoulli's equation between 1 and 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + losses$$

$\because$ Flow is inviscid or ideal flow, losses are neglected

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + (Z_2 + Z_1)$$



From continuity equation

$A_1V_1 = A_2V_2$

given, $A_1 = 2A_2$

$\therefore$ $V_2 = 2V_1$

$$\frac{P_1 - P_2}{\rho g} = \frac{3V_1^2}{2g} + 0.15$$

$$\frac{P_1 - P_2}{\rho g} = -\frac{1.5}{10} + 0.15$$

$P_1 - P_2 \Rightarrow 0.3 \times 10$

$P_1 - P_2 = 3Pa$

9. [1.21 m]

Applying Bernoulli equation to points 1 & A

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A$$

Taking point A as the reference/Datum

Now

P_A = vapour pressure for critical case because the velocity is high at point 'A' [$\therefore$ Pressure is lesser compared to other points]

Using that

$$\frac{V_A^2}{2g} = \frac{95.48 \times 10^3 - 4 \times 10^3}{9.81 \times 10^3} + 1.5$$

$$\frac{V_A^2}{2g} = 10.825 \text{ m}$$

V_A = 14.57 m/sec

Now applying Bernoulli's equation between points (1) & (2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Taking point 2 as datum

$$\frac{P_{stm}}{\gamma} + 0 + 1.5 + L = \frac{P_{atm}}{\gamma} + \frac{V_2^2}{2g} + 0$$

$$\Rightarrow L - \frac{V_2^2}{2g} - 1.5$$

But given $V_2 = \frac{V_A}{2} = \frac{14.57}{2}$

$V_2 = 7.285 m/\sec$

Using (1)

$L = 1.205m \approx 1.21m$

10. $\left[\dfrac{u^2 - v^2}{2g}\right]$

$$\frac{u^2}{2g} = h + \frac{v^2}{2g}$$

$$h = \frac{u^2 - v^2}{2g}$$

11. [B]

The height reached by stream would be equal to the velocity head.

12. [A]

Bernoulli's equation is valid for steady, inviscid, incompressible flow along a stream line.

13. [1.47 kN]

The cross-sectional area of the jet is

$$a = \frac{\pi \times 0.012^2}{4}$$

$= 1.13 \times 10^{-4}$ m^2

The discharge from each nozzle is

$$q = \frac{Q}{2}$$

$= 7.5 \times 1^{-4}$ m^3/s

Jet velocities are

$$v_1 = v_2 = \frac{q}{a}$$

= 6.63 m/s

The radius of arms are

r = .25 m

R = 0.45 m

The inclination angle is

β = 180 – 60

= 120°

The moment of momentum is

$$T = -pqr\left(u_1 + v_1\cos\beta\right) + pqR\left(u_2 + v_2\cos\beta\right)$$

$$= -pqr\left(r\omega + v_1\cos\beta\right) + pqR\left(R\omega + v_2\cos\beta\right)$$

When friction is zero, T = 0, hence

$$pqr\left(r\omega + v_1\cos\beta\right) = pqR\left(R\omega + v_2\cos\beta\right)$$

Therefore,

$$\omega = \frac{\left(v_2 R - v_1 r\right)\cos\beta}{-\left(R^2 - r^2\right)}$$

$$= -\frac{v_2\left(R - r\right)\cos\beta}{\left(R^2 - r^2\right)}$$

$$= -\frac{v_2\cos\beta}{\left(R + r\right)}$$

Substituting the values,

ω = 7.435 rad/s

= 70.85 rpm

Torque required the sprinkler stationary (ω = 0) is given by

$$T = -pqv_2\cos\beta\left(R + r\right)$$

= 1.47 kN

14. [10.19 kN]

Given that discharge is Q = 0.06 m^3/s. At section 1, the diameter of pipe is d_1 = 0.15 m, therefore, velocity of flow is

$$v_1 = \frac{4Q}{\pi d_1^2} = 3.395 \text{ m/s}$$

At section 2, the diameter of pipe is d_2 = 0.05 m, therefore, velocity of flow is

$$\frac{v_2^2 - v_1^2}{2g} = \frac{p_1}{\rho g}$$

p_1 = 461.19 kPa

Considering the bend as free body, the momentum equation in x-direction is

$$-p_1 A_1 + R_x = \rho Q[v_2 - \left(-v_1\right)]$$

$$R_x = 10.19 \text{ kN}$$

15. [B]

Power transmitted is maximum when the loss of head due to friction is one-third of the total head supplied.

16. [20m]

$$0.9 = \frac{72}{H}$$

$\therefore$ H = 80 m

$\therefore$ Pipe friction = 100 – 80 = 20 m

17. [D (1.40 to 2.00)]

18. [B]

Specific speed, $N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$

$$N_S = \frac{100\sqrt{6000}}{(25)^{5/4}}$$

$$N_S = 138.564$$

The specific speed of Francis turbine ranges form 75 to 305 for all practical purposes.

19. [C]

Tubular or bulb turbine are designed into the water delivery tube. A large bulb is centred in the water pipe which holds the generator, wicket gate and runner. Tubular turbine are a fully axial design.

20. [20m]

$$0.9 = \frac{72}{H}$$

$\therefore$ H = 80m

$\therefore$ Pipe friction = 100 – 80 = 20m

21. [C] Relative velocity remain constant

22. (B) Kaplan Turbine

23. [88.88 (88-90)]

Given data;

$$\beta_i = 90°$$

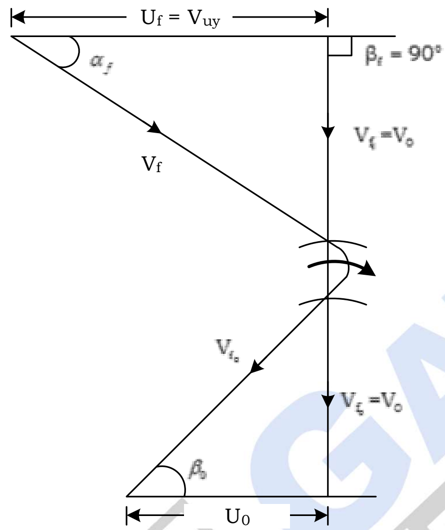
$$V_{wo} = 0$$

$$V_{t_f} = V_{t_o} = \frac{u_f}{2}$$

Blade efficiency of the runner,

$$\eta_H = \frac{V_{wi}u_i}{gH} = \frac{u_i^2}{gH}$$

$$\because V_{wi} = u_i$$



By energy balance equation,

$$pQgH = pQV_{wi}u_i + \frac{pQV_0^2}{2}$$

$$gH = u_f^2 + \frac{V_{f0}^2}{2} = u_i^2 + \frac{u_i^2}{8}$$

$$gh = \frac{9}{8}u_i^2$$

$$\therefore \quad \eta_H \frac{u_i^2}{\frac{9}{8}u_i^2} = \frac{8}{9} = 0.8888$$

= 88.88% = **89%**

24. [3.417(3.40 – 3.45)]

H = 500 m; D = 0.3 m;

L = 3500 m; f = 0.006

$$h_f = \frac{H}{3} = \frac{500}{3} = 166.7 \text{ m}$$

$$\Rightarrow \quad h_f = \frac{4fLv^2}{D \times 2g}$$

$$\Rightarrow \quad 166.7 = \frac{4 \times 0.006 \times 3500 \times v^2}{0.3 \times 2 \times 9.81}$$

$\Rightarrow$ v = 3.417 m/s

25. (A)

Given: H_m = 1.5 m; H_p = 30 m;

N_m = 450 rpm $\frac{D_m}{D_p} = \frac{1}{5}$

$$\frac{D_m}{D_p} = \frac{N_p}{N_m}\sqrt{\frac{H_m}{H_p}}$$

$$\Rightarrow \quad \frac{1}{5} = \frac{N_p}{450}\sqrt{\frac{1.5}{30}}$$

$$\Rightarrow \quad N_p = 90\sqrt{20} = 402rpm$$

Now, comparing specific speeds of model and prototype,

$$\Rightarrow \quad \frac{N_p\sqrt{P_p}}{H_p^{5/4}} = \frac{N_m\sqrt{P_m}}{H_m^{5/4}}$$

$$\Rightarrow \quad \frac{402\sqrt{P_p}}{(30)^{5/4}} = \frac{450\sqrt{5}}{(1.5)^{5/4}}$$

$$\Rightarrow \quad P_p = 11208\text{kW} \simeq 11.2\text{MW}$$

26. (D)

$$V_1 = \sqrt{2gh_1} = 7.0035 \text{ m/s},$$

$$m = \rho AV_1 = 70.035 \text{ kg/s}$$

$$V_2 = \sqrt{2g(h_1 + h_2)} = 12.13 \text{ m/s}$$

T_1 is tension in rope A,

$$\Rightarrow \quad T_1 = \rho AV_1^2 = 1000 \times 0.01 \times (7.0035)^2 = 490 \text{ N}$$

Now $T_2 - T_1 = m(V_2 - V_1)$

= 70.035 (12.13 – 7.0035)

$T_2 - T_1 = 359.03$

So, T_2 = **849.52 N**

27. (D)

Speed of turbine, N = 450 rpm

Head, H = 120 m

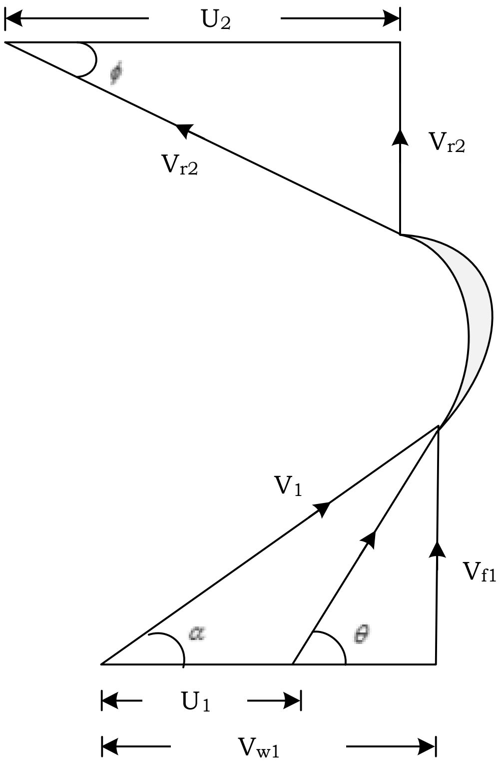
Diameter at inlet, D_1 = 120 cm = 1.2 m

α = 20°

θ = 60°

$V_{W2} = 0$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 \text{ m/s}$$



Now, from inlet velocity triangle,

$\Rightarrow \quad V_{r1}\sin\theta = V_{f1}$

$\Rightarrow \quad V_{r1}\cos\theta = V_{w1} - u_1$

Now, $\tan\theta = \dfrac{V_{f1}}{V_{w1} - u_1}$

$\Rightarrow \quad \tan\theta = \dfrac{V_{w1}\tan\alpha}{V_{w1} - u_1} \qquad \left[\because \tan\alpha = \dfrac{V_{f1}}{V_{w1}}\right]$

$\Rightarrow \quad \tan\theta = \dfrac{V_{w1}\tan 20^\circ}{V_{w1} - u_1}$

$\Rightarrow \quad V_{w1} = 35.79\text{ m/s}$

Hydraulic efficiency, $\eta_h = \dfrac{Vw1u_1}{gH} = \dfrac{35.79\times 28.27}{9.81\times 120}$

= 0.8595 = **85.95%**

28. [1.21]

P_A = Vapour pressure = 4.00 kPa (abs)

$P_1 = P_2$ = atmospheric pressure

= 95.47 kPa(abs)

By applying Bernoulli's equation to point 1 and A

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A$$

$$\left(\frac{95.48}{9.81}\right) + 0 + 1.5 = \left(\frac{4.00}{9.81}\right) + \frac{V_A^2}{2g} + 0$$

$$\frac{V_A^2}{2g} = \frac{95.48 - 4.00}{9.81} + 1.5 = 10.82\text{ m}$$

$$v_A = (2\times 9.81\times 10.825)^{1/2} = 14.57\text{ m/sec}$$

$$v_2 = \frac{V_A}{2} = \frac{14.57}{2} = 7.28\text{ m/sec}$$

By applying Bernoulli's equation to point 1 and 2, with datum at point 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{95.48}{9.81} + 0 + (L + 1.50)$$

$$= \left(\frac{95.48}{9.81}\right) + \frac{(7.28)^2}{2\times 9.81} + 0$$

L = 1.21

29. 0.316(0.3 – 0.35)

Q = 1000 litre/s = 1 m³/s

$$A_f = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}\times(0.5)^2$$

$$v_1 = \frac{Q}{A_1} = \frac{1}{\frac{\pi}{4}\times(0.5)^2} = 5.09\text{ m/s}$$

$$A_2 = \frac{\pi}{4}\times d_2^2 = \frac{\pi}{4}\times(0.7)^2$$

$$V_2 = \frac{Q}{A_2} = \frac{1}{\frac{\pi}{4}\times(0.7)^2} = 2.6\text{ m/s}$$

Head loss = $\dfrac{(V_1 - V_2)^2}{2g} = \dfrac{(5.09 - 2.6)}{2\times 9.81} = 0.316m$

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