ANSWER KEY

1. B	**2.** B	**3.** B	**4.** B	**5.** D	**6.** C	**7.** B	**8.** C	**9.** D	**10.** D
11. C	**12.** C	**13.** A	**14.** A	**15.** B	**16.** C	**17.** C	**18.** C	**19.** B	**20.** B
21. B	**22.** B	**23.** D	**24.** A	**25.** A	**26.** B	**27.** A	**28.** C	**29.** D	**30.** A
31. A	**32.** A	**33.** D	**34.** B	**35.** D	**36.** B	**37.** C	**38.** C	**39.** C	**40.** A

SOLUTIONS

23. $u = 3\times y^2,\; v = 2\times y,\; w = (2zy + 3t)$

At point (1,2,1) and t = 3,

$u = 12,\; v = 4,\; w = 13$

In vector form $\Rightarrow 12\hat{i} + 4\hat{j} + 13\hat{k}$

24.

$$V = \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3\times y^2 & 2\times y & (2zy+3t) \end{vmatrix}$$

$$V = \frac{1}{2}\left[\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)i + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)j + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)k\right]$$

$$= \frac{1}{2}\left[(2z-0)i + (0+0)j + (2y-6xy)k\right]$$

$V = zi + (2y - 4xy)\hat{k}$

At, $(1,2,1)$, $\Rightarrow V = i + (4-8)k$

$V = i - 4\hat{k}$

25. Vorticity of fluid element $= 2(\text{Rotectional Flow})$

$\Rightarrow$ Vorticity of fluid element $= 2(i - 4k)$

$\therefore\; V = 2\hat{i} - 8\hat{k}$

26. $u = 4x^3, v = -10x^2y, w = 2t$

At, $(2,1,3)$ *and* $t = 1$

$u = 32, v = -40, w = 2$

$|(V)| = \sqrt{u^2 + v^2 + w^2}$

$= \sqrt{32^2 + 40^2 + 2^2}$

51.264 *mts*

27. For any incompressible fluid flow

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$,

$\frac{\partial u}{\partial x} = 2x, \frac{\partial v}{\partial y} = +2xy - z^2 + x$,

So, $2x + 2xy - z^2 + x + \frac{\partial w}{\partial z} = 0$,

$\Rightarrow w = -3xz - 2xyz + \frac{z^3}{3} + const$

28. $u = \frac{\partial ¤}{\partial x} = \frac{-y^3}{3} - 2x + x^2y$,

$v = \frac{+\partial ¤}{\partial y} = -xy^2 + \frac{x^3}{3} + 2y$

29 $u = \frac{\partial ¤}{\partial x} = 10x = 40$, $v = \frac{\partial ¤}{\partial y} = -10y = -50$

30. $¤ = x(2y-1)$

$u = \frac{\partial ¤}{\partial x} = (2y-1)$

$v = \frac{\partial ¤}{\partial y} = 2x$

Also, $u = \frac{\partial \cdot}{\partial y} = 2y - 1$

$\cdot = y^2 - y + c \Rightarrow \frac{\partial \cdot}{\partial x} = \frac{\partial c}{\partial x}$

$v = \frac{\partial \cdot}{\partial x} = 2x \Rightarrow \frac{\partial c}{\partial x} = 2x$

Or, $c = x^2$

So, $\cdot = y^2 - y + x^2$

31. $u = \frac{\partial \cdot}{\partial y} = 2x = 4$

$v = \frac{-\partial \cdot}{\partial x} = 2y = 6$

$|(V)| = \sqrt{u^2 + v^2} = \sqrt{16 + 36} = \sqrt{52}$

$= 7.2\ unit/\text{sec}$.

32. $u = \frac{\partial \cdot}{\partial y} = 2x$

$v = \frac{-\partial \cdot}{\partial y} = 2y$, *also* $u = \frac{\partial ¤}{\partial x} = 2x$

$\Rightarrow\quad ¤ = x^2 + c$,

Or, $\frac{\partial ¤}{\partial y} = \frac{\partial c}{\partial y}$

$\frac{\partial ¤}{\partial y} = v = 2y \Rightarrow \frac{\partial c}{\partial y} = 2y \Rightarrow c = y^2$

So, $¤ = x^2 + y^2$

33. $u = \frac{\partial \cdot}{\partial y} = 2y = 4\ ,\ v = \frac{-\partial \cdot}{\partial y} = 2x = -2$

34. Shear strain rate = $\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$\frac{\partial v}{\partial x} = -20xy\ ,\ \frac{\partial u}{\partial y} = 0$

$\Rightarrow$ *Shear* strain rate = $\frac{1}{2}(-20xy - 0)$

$= -10xy$

35. For steady state in compressible flow,

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\ ,$

$\Rightarrow ª y^3 - 2xy + 2xy - 3y^2 = 0 \Rightarrow y^3(ª - 3) = 0$

$\therefore ª = 3$

36. To check incompressible flow

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\ \ and\ \ \frac{\partial w}{\partial z} = 0$

$For\ A \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \neq 0$

$For\ B \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Hence, B only is incompressible flow.

37. $u = \frac{\partial \cdot}{\partial y}\ \ and\ \ v = -\frac{\partial \cdot}{\partial y}$

differentiating · we get u= $\frac{2y}{x^2 + y^2}, v = \frac{-2x}{x^2 + y^2}$

38. $A = 0.5 - 0.2x$

$dA/dx = 0.2\ \ and\ \ u = \frac{dQ}{dA}$

$Q = vA$

$\frac{dQ}{dt} = v\frac{dA}{dt} + A\frac{dv}{dt}$

at $x = 0\ \ A = 0.5$
and $Q = A \times v$
$\Rightarrow 0.5 = 0.5 \times v$
$v = 1m/s$

$\Rightarrow\ 0.2 = 1\frac{dA}{dx}.\frac{dx}{dt} + 0.5 \times acc$

$\frac{dx}{dt} = v = 1m/s$

$= ac_C = \frac{0.4}{0.5} = 0.8$

39. $d\cdot\ = -Vdx + udy = -8xdx + 2dy$

$\cdot\ = -4x^2 + 2y$

40. For a potential flow both $\nabla^2 ¤\ \ and\ \ \nabla^2 \cdot\ \ is\ \ zero$

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